Hamromaster
सुनौलो भविष्य हातमा **Hamromaster** को साथमा

Hamromaster
COMPLETE NOTES

For Hamromaster Online Classes

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Hamromaster Hamromaster

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Particle nature of wave:-

Einstein photoelectric effect: The process of emission of electron from a metal surface by the incidence of high frequency radiation

Einstein photoelectric eqn:

$$hf = \phi + KE$$

where,

hf : energy of incident radiation

ϕ : work formula

KE = KE of emitted electron

$hf > \phi$ only electron emission possible

Photon: Quantum of light, smallest packet of light. It has no rest mass

Energy of mass $mc^2 = hf$

$$\therefore m = \frac{hf}{c^2}$$

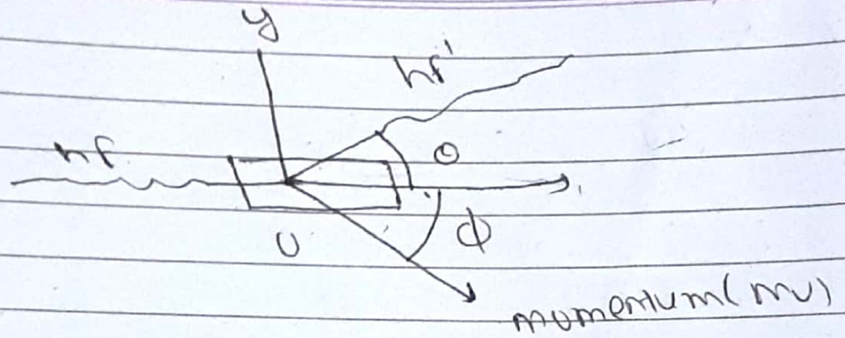
momentum of photon (h/λ).

Photon is not deviated by both electric and magnetic field.

VVI 5 marks

Compton's effect:

Whenever a photon of certain energy is incident upon a matter. The photon interact with matter as a result. Some amount of energy is used by the photon and the same amount of energy is gained by the matter such kind of elastic interaction is called Compton's effect.



Let us consider a photon of energy hf is incident upon a matter such that the photon interact with the matter electron having rest mass energy m_0c^2. After the interaction (collision) photon's with energy hf' recoils along the direction θ with x-axis and the electron scattered with energy mc^2 momentum (mv) along the direction making an angle ϕ with the x-axis.

Now, using principle of conservation of energy.

$$hf + m_0c^2 = hf' + mc^2 \quad \text{---(i)}$$

Where,

f' = frequency of recoil photon

m_0 = rest mass of electron

m = relativistic mass of photon

Also, using the principle of conservation of linear momentum.

Momentum along x-axis

$$\frac{hf}{c} = \frac{hf'}{c}\cos\theta + mv\cos\phi$$

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$\Rightarrow h(f - f'\cos\theta) = mcv\cos\phi$ — (i)

Along y-axis,

$0 = \frac{hf'}{c}\sin\theta - mv\sin\phi$

$hf'\sin\theta = mcv\sin\phi$ — (ii)

Squaring and adding (i) and (ii)

$h^2(f - f'\cos\theta)^2 + h^2 f'^2 \sin^2\theta = m^2c^2v^2\cos^2\phi + m^2c^2v^2\sin^2\phi$

$\Rightarrow h^2(f^2 - 2ff'\cos\theta + f'^2) = m^2c^2v^2$

from eq^n (i)

$mc^2 = h(f - f') + m_0c^2$

Squaring,

$m^2c^4 = h^2(f - f')^2 + 2h(f - f')m_0c^2 + m_0^2c^4$

$\Rightarrow h^2f^2 - 2h^2ff' + h^2f'^2 + 2h(f - f')m_0c^2 + m_0^2c^4 = m^2c^4$ — (v)

Subtracting (iv) from (v)

$m^2c^2(c^2 - v^2) = -2h^2ff' + 2ff'\cos\theta + 2h(f - f')m_0c^2 + m_0^2c^4$

$\Rightarrow \left(\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right)^2 c^2(c^2 - v^2) = 2h^2ff'(\cos\theta - 1) + 2h(f - f')m_0c^2 + m_0^2c^4$

$\Rightarrow 0 = 2h^2ff'(\cos\theta - 1) + 2h(f - f')m_0c^2$

$\Rightarrow 2h^2ff'(1 - \cos\theta) = 2h(f - f')m_0c^2$

$\Rightarrow hff'(1 - \cos\theta) = (f - f')m_0c^2$

$\Rightarrow f - f' = \frac{hff'}{m_0c^2}(1 - \cos\theta)$

$\Rightarrow \frac{c}{\lambda} - \frac{c}{\lambda'} = \frac{h\frac{c}{\lambda}\cdot\frac{c}{\lambda'}}{m_0c^2}(1 - \cos\theta)$

Pair production : Photon and positron

$$c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{h}{m_0\lambda\lambda'}(1-\cos\theta)$$

$$\Rightarrow (\lambda'-\lambda) = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Rightarrow \Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) \quad \text{--- (vi)}$$

Also,

$$\Delta\lambda = \frac{2h}{m_0c}\sin^2\theta/2 \quad \text{--- (vii)}$$

The equation (vi) and (vii) gives expression for wavelength is called Compton's shift

Special case

when $\theta = 0$

$\Delta\lambda = 0$ — no change in energy.

when $\theta = \pi/2$

$\Delta\lambda = \frac{h}{m_0c}$ — maximum change in energy

5 marks

Pair Production:

When energy of incident photon is equal to 1 MeV then the photoelectric effect is occurred if the energy of incident photon is comparable to 1.02 MeV twice of m_0c^2 then the emission of pair production takes place but further energy

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of incident photon about 1.5 Mev. then compton's effect occured.

Interaction of γ rays with matter

whenever a γ-radiation incident upon a matter then may be 3 cases

i) It can be penetrate the section of matter without any interaction

ii) It can interact with matter and it's energy completely absorb by the matter.

iii) It can interact with matter and scattered from it's original direction taking some energy

let us consider a γ-ray (x-ray) photon with energy hf and initial intensity I_0 is incident on a matter. After passing the distance 'x' on a matter it's intensity becomes I. then rate of change in intensity of γ ray (or x-ray) with distance directly proportional with the intensity of γ-ray photon at that time.

$$\frac{dI}{dx} = -\mu I$$

where 'μ' is linear absorption coefficient having an unit of per meter and -ve sign means intensity of photon decreasing on passing matter. Now, $\frac{dI}{I} = -\mu dx$

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$\Rightarrow [\ln I]_{I_0}^{I} = \mu [x]_0^x$

[$\because$ taking integration from I_0 to I on passing the thickness 0 to x.

or, $\ln I - \ln I_0 = -\mu x$

$\Rightarrow \ln\left(\frac{I}{I_0}\right) = -\mu x$

$\Rightarrow \frac{I}{I_0} = e^{-\mu x}$

$\Rightarrow I = I_0 e^{-\mu x}$

$\Rightarrow \boxed{I = I_0 e^{-\mu x}}$

This is the required expression for decrease in intensity of γ-ray on passing through matter.

From this relation it is cleared that intensity of γ-ray (or x-ray) decrease exponentially with distance.

3 marks

Half value thickness:

It is a width of a material required to reduce the intensity of γ-ray to half ~~of~~ of it's original intensity (initial intensity)

We have,

$I = I_0 e^{-\mu x}$

At half value thickness, $I = I_0/2$

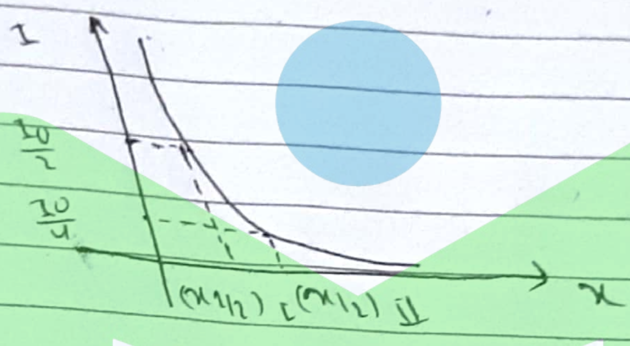
$\Rightarrow \frac{I_0}{2} = I_0 e^{-\mu x_{1/2}}$

$\Rightarrow 2 = e^{\mu x_{1/2}}$

$\Rightarrow \mu x_{1/2} = \ln 2$

$\Rightarrow \boxed{x_{1/2} = \frac{0.693}{\mu}}$

Graphically,



Q. The linear absorption coefficient for a gamma ray in a lead is 78 per meter. Find thickness of lead required to reduce half the intensity of beam of such gamma ray.

Soln.

$x_{1/2} = \frac{0.693}{\mu} = \frac{0.693}{78}$

$\therefore x_{1/2} = 8.88 \times 10^{-3}\, s^{-1}$

Q. Find most absorption coeffient of density $8930\, kg/m^3$. If 1.05 mm of copper reduces x-ray beam to 0.075 of it's original intensity.

Sol.

$\rho = 8930\, kg/m^3$

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$1\ \text{Pm} = 10^{-12}\,m$

$\boxed{1\,\text{Pm} = 10^{-12}\,m}$

$I = 0.075\, I_0$

$I = I_0\, e^{-\mu_m \rho x}$

$\Rightarrow 0.075\, \cancel{I_0} = \cancel{I_0}\, e^{-\mu_m \rho x}$

$\Rightarrow -\mu_m \rho x = -2.59$

$\Rightarrow \mu_m \times 8930 \times 1.05 \times 10^{-3} = 2.59$

$\Rightarrow \mu_m = 0.276$ per kg

Q. An x-ray is found to have wavelength of $0.124\, A^0$ and undergoes Compton's effect from carbon brick. Calculate the wavelength when it scattered through 180^0

Sol.

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Rightarrow \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Rightarrow \lambda' = \lambda + \frac{h}{m_0c}(1-\cos\theta)$$

Q. X-ray of wavelength 10 Pm scattered through target. Calculate the wavelength when it is scattered through 45^0

Sol.

$$\boxed{\lambda' = \lambda + \frac{h}{m_0c}(1-\cos\theta)}$$

Nuclear physics:

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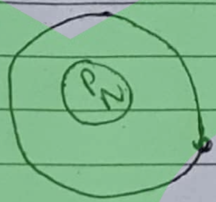
Nuclear constituent:

constituent of nucleus are proton and neutron mass of proton is equal to 1.67×10^{-27} kg and charge is +ve and the mass of neutron is 1.67×10^{-27} kg and no charge.

proton and neutron both show the spin motion. The proton has magnetic moment while neutron has no magnetic moment

$$\boxed{\mu = \frac{eh}{4\pi m}}$$

Nuclear property
- Nuclear charge
- Nuclear mass
- Nuclear size
- Nuclear density



Nucleus contain proton and neutron. Proton has +ve charge (+1e) while neutron has no charge. So the net charge of nucleus is charge carried by proton inside the nucleus.

If there are Z protons inside the nucleus then nuclear charge.

$$q = +Ze$$

where Z is atomic number (charge number)

$1 fm = 10^{-15} m$

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Nuclear mass:
Nucleus contain proton and neutron. Both have finite mass. So, both proton and neutron contribute for nuclear mass.

If a nucleus contains Z protons each of mass m_p and N neutrons each of mass m_n. Then, total nuclear mass,

$$M = Z m_p + N m_n$$

where, $N = A - Z$

where A is mass number

So, total nuclear mass

$$M = Z m_p + (A-Z) m_n$$

Nuclear size:
Nuclear size refers to the volume of the nucleus since nucleus is spherical so volume is given by

$$V = \frac{4}{3}\pi R^3$$

Also, we have,

$$R = R_0 A^{1/3}$$

where, R_0 is cut off radius of nucleus

$$R = 1.2 fm$$

then volume,

$$V = 4/3 \pi R_0^3 A$$

All are fixed except A so,

$$V \propto A$$

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Nuclear density:

Nuclear density is the ratio of nuclear mass and nuclear volume ie

$$\rho_N = \frac{mass}{volume} = \frac{Zm_p + (A-Z)m_n}{4/3 \pi R_0^3 A}$$

$$= \frac{A m_p}{\frac{4}{3}\pi R_0^3 A} \quad [\because m_n \simeq m_p]$$

$$\rho_N = \frac{m_p}{4/3 \pi R_0^3}$$

Here, all quantities are fixed So, nuclear density is constant for all nucleus

$\rho_N \sim 2\times10^{17}$ kg/m^3 which is 10^{14} times greater than the density of water.

Nuclear Spin and magnetic moment.

Nuclear spin is due spining motion of proton and neutron both but nuclear magnetic moment is due to proton inside the nucleus which is in the order of $\frac{eh}{4\pi m}$

mass defect:

This is difference in mass of nucleus and it's constituents ie.

mass defect = mass of Constituent − mass of nucleus

or, $\Delta m = Zm_p + (A-Z)m_n - M$

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Δm is +ve for stability of nucleus

Binding energy:
Energy that binds nucleons inside the nucleus OR
It is energy associated with mass defect.

Binding energy (BE) = $\Delta m c^2$ in J
(kg)
= $\Delta m \times 931$ MeV
(amu)

Here,
$BE = [Zm_p + (A-Z)m_n]c^2$ — in J
and,
(amu)
$BE = [Zm_p + (A-Z)m_n]\,931$ MeV

Mirror nuclei:
Two nuclei in which number of proton of one nucleus is equal to number of neutron of other nucleus is called mirror nuclei. In case of mirror nuclei mass number must be same eg

$_8O^{15}$ and $_7N^{15}$ in $_8O^{15}$

Proton = 8 neutron = 7 in $_7N^{15}$

Proton = 7
neutron = 8
Here, these are mirror nuclei

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Electron cannot exist inside the nucleus

Taking uncertainity of nucleus inside the nucleus in the order of 10^{-15} m (size of nucleus) [uncertainity of electron if electron exists inside the nucleus]
ie $\Delta x \sim 10^{-15}$ m

then,
Using Heisenberg uncertainity principle,

$$\Delta x \, \Delta p = \frac{h}{2\pi}$$

$$\Delta p = \frac{h}{2\pi \Delta x}$$

Now, $KE = \frac{p^2}{2m} = 20 \text{ meV}$

but experimental value of energy of electron is in the order of (2-3) MeV. The speed of electron with energy 20 MeV is greater than the speed of light. This means it is not possible to exist electron inside the nucleus.

Binding energy per nucleon:

Binding energy per nucleon is the ratio of binding energy of nucleus to it's mass number ie

$$\frac{BE}{A} = \text{Binding energy per nucleon}$$

BE per nucleon gives the stability of the nucleus.

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Nuclear stability

Experimentally, most stable nuclei are found to have number of proton and number of neutron are as follows.

no of neutrons	proton number	no of total nuclei
even	even	160
even	odd	56
odd	even	52
odd	odd	4
		272

From this chart it is clear that the most stable nuclei are those in which even number of proton and even number of neutron. This result prefer that even-even combination of proton and neutron.

The nuclei for which even number of proton and odd no of neution and vice-versa are less stable then the nuclei with even-even combination.

The nuclei for which odd-odd combination of neutron and proton are very less stable so nature doesnot prefer this combination.

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Nuclear quadrapole moment:

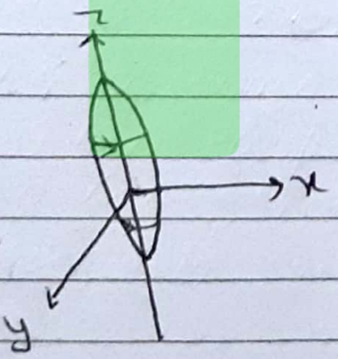
The nuclear quadrapole moment gives the disturbed are of nucleus from the spherically symmetric position. If the nucleus is in spherically symmetric form then the value of quadrapole moment is zero, mathematically, the quadrapole moment,

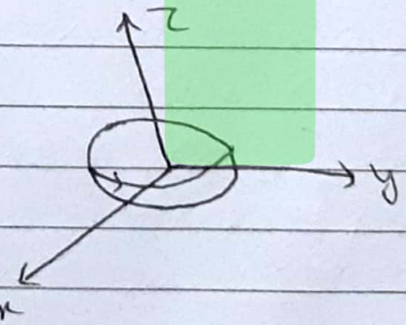
$$Q = \frac{1}{e} \int (3z^2 - r^2) \, dz$$

wher, $r^2 = x^2 + y^2 + z^2$
and dz is volume integration

If the nucleus is spherically symmetric about all axis then $x = y = z$ and then the value of $Q = 0$. If the nucleus is oriented about z-axis the value of Q is +ve and the shape of nucleus is called prolate.

If the nucleus is oriented about x and y-axis and the shape of nucleus is oblate





X-ray:

Electromagnetic wave

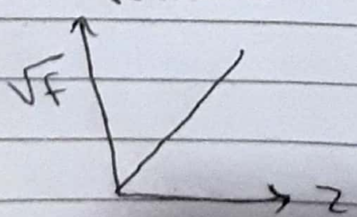
Radio wave
micro wave
infrared
visible
UV
x-ray
γ-ray

frequency and energy (increases)

wavelength decreases

X-ray is an electromagnetic wave with high frequency and less wave length in the order of (0.2-100) A^0

Moseley law:
Moseley conducted many experiments to established the relation between frequency of emitted x-ray and the atomic number of target atom but no any remarkable result could acheive. Finally he plotted the graph between square root of frequency of emitted x-ray and atomic number of target atom. The graph is obtained as follows:



on the basis of this experiment moseley

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formulate a rule This is called mosley law

Statement: It states that the square root of frequency of x-ray emitted from a target atom is directly proportional with atomic number of target atom i.e

$$\sqrt{f} \propto Z$$

$$\sqrt{f} = a(Z-b)$$

where 'a' is proportionality constant and 'b' is screening constant

Theoritical explanation of mosley law.

we have from Bohr's theory,

$$E_n = -\frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2} \quad (\text{for H-like atom})$$

$$E_{n_1} = -\frac{me^4 Z^2}{8\varepsilon_0^2 n_1^2 h^2}$$

$$E_{n_2} = -\frac{me^4 Z^2}{8\varepsilon_0^2 n_2^2 h^2}$$

$$\therefore E_{n_2} - E_{n_1} = \frac{me^4 Z^2}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

using Bohr's postulates

$$hf = \frac{me^4 Z^2}{8\varepsilon_0 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or, $f = \frac{me^4 z^2}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\therefore \boxed{f \propto z^2}$

Continuous and Characteristic x-ray

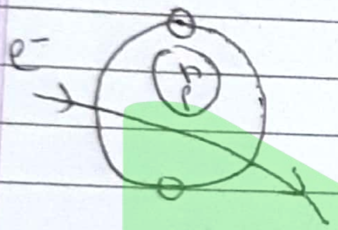
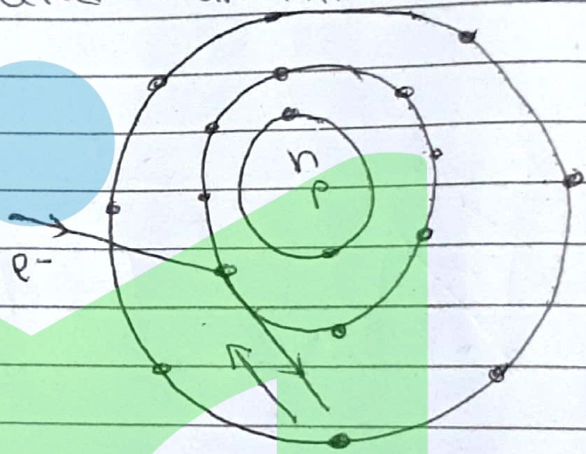


fig: continous x-ray



emits photon equivalent to characteristic x-ray

fig: characteristic x-ray

The x-rays are emitted by the interaction of fast moving electron with target atom having very high atomic number.

The continuous x-ray are emitted by the interaction of fast moving electron with ion core of target atom. when fast moving electron interact with ion core it get deacceleration and the deaccelerated electron emits photon. This is equivalent to continous x-ray

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When fast moving electron with energy equivalent to binding energy of target atom is incident on a target atom. Any of the electrons of target atom may be ejected by the interaction with fast moving electron. A vacancy is created there and the vacancy is fulfilled by the transition of electron from any of the outer orbits with the emission of radiation. These radiations are equivalent to characteristic x-ray.

If K-shell electron is ejected by the interaction with fast moving electron. The vacancy in K-shell is fulfilled by the transition of electron from any of the outer orbits. When ~~vata~~ vacancy fulfilled by L-electron the emitted radiation is called K_α line. If the vacancy is fulfilled by electron in M-orbit then the radiation (spectra) is said to be K_β line.

By the similar way the 1st line and 2nd line due to the ~~radiation~~ transition of electron from n=4 and 3 to n=2 are called L_β and L_α lines.

Bragg's Spectrometer (X-ray)

Spectrometers are device which are used to analyze spectra.

Bragg's spectrometer is one which is used to analyze X-ray spectra. The construction of Bragg's spectrometer is similar to that of the optical

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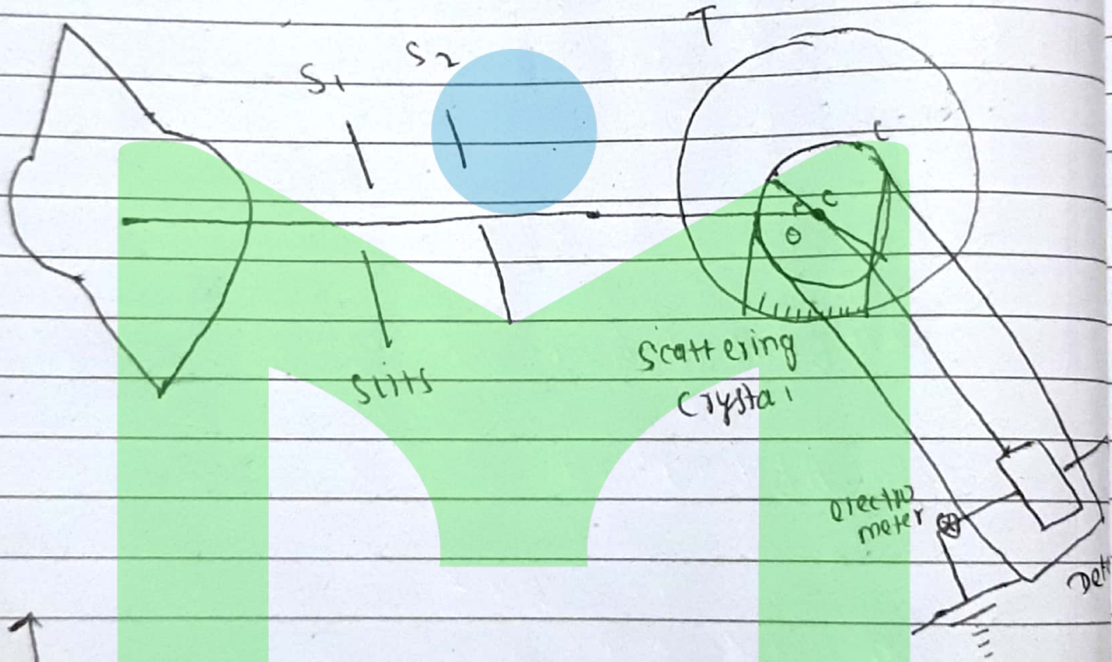
Spectrometer: The essential parts of Bragg's spectrometers are:

(i) Source of X-ray (Source)

(ii) Crystal held on round table Scattered |

(iii) The detector



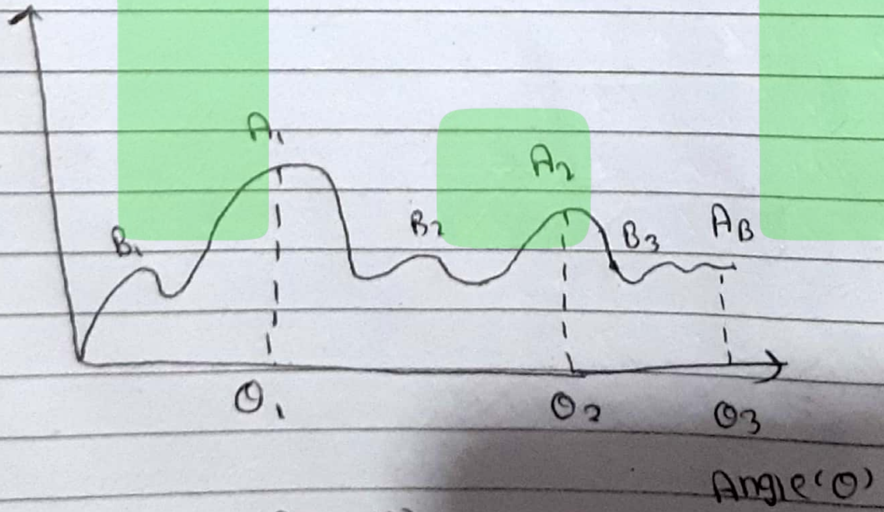


fig (2)

A beam of X-ray from the source is passed through two narrow slit S_1 and S_2 to make the beam

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fine. After making fine the beam is incident on a crystal of around table 't' such that the x-ray gets diffracted. The diffracted x-ray then enter to ionization chamber that contains electrometer which measure intensity of diffracted x-ray in terms of current.

The x-rays are incident to the crystal with different glancing angle. By varying glancing angle θ, the corresponding intensity of diffracted x-ray is measured with the electrometer.

The graph is plotted between intensity and glancing angle, the nature of graph shown in fig (1) which is called Bragg's (x-ray) spectrum. The prominent peaks A_1, A_2 - corresponds to the x-ray of wavelength λ. The glancing angle $\theta_1, \theta_2, \theta_3$ - - are corresponding to the peaks A_1, A_2, A_3 -

from the x-ray spectrum we can find that
$\sin\theta_1 : \sin\theta_2 : \sin\theta_3 : \cdots = 1 : 2 : 3 : \cdots$

The indicates that the prominent peaks A_1, A_2 - - are primary, secondary - - maximas for x-ray of wavelength λ. The another peaks B_1, B_2, - are the first, second - - order maxima for the wavelength λ_2.

measurement of wavelength
we have Bragg's equation ie

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$2d\sin\theta = n\lambda$

where 'd' is lattice spacing and θ is glacing angle for orders of diffraction.
λ = wavelength of x-ray

If the value of 'd' is known then the value of θ can be obtained from Bragg's spectrum. Then the wavelength of x-ray can be measured

vvi Importance of Bragg's law:

i) Bragg's law can be used to study the structure of crystal

ii) It also has medical use (X-ray diffraction)

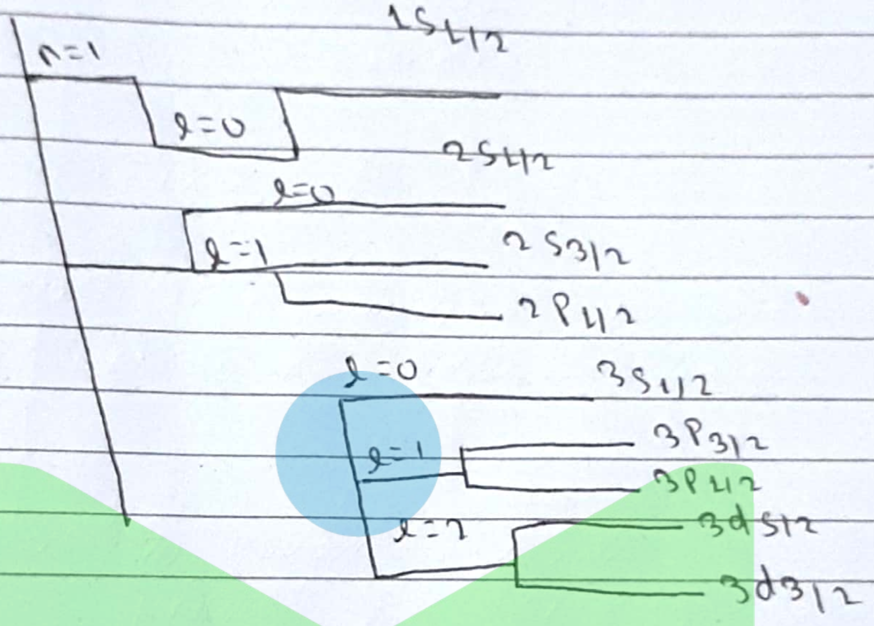
vvi Fine structure of X-ray:-

The characteristics X-ray spectrum is not single but it consists of large number of closely packed lines (fine structure) they are called fine structure of X-ray

The splitting of ℓ-line into 3 lines and m line into 5 lines. They are fine structure of X-ray. The fine structure of X-ray can be explained due to the following two effects

i) Screening effect
ii) The relativistic variation of mass with velocity

The fine structure of x-ray is shown in following figure:



Screening effect:

The eight electron present in L-shell of an atom finds two electrons in K-shell. That reduces the effect of nuclear charge & similar to be in the case of electron present in M-shell. The effect of reducing nuclear charge due to the presence of inner electron is called screening effect.

Q. The 1st order Bragg's reflection occurs at an angle of 10°. When x-ray of wavelength 9.8 nm is used in NaCl crystal. What is the spacing between principle planes of crystal.

Soln.

$n = 1$

$\theta = 10^\circ$

$\lambda = 9.8\,nm = 9.8\times10^{-9}\,m$

$\therefore d = ?$

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we know that,

$$2d\sin\theta = n\lambda$$

$$\Rightarrow 2d\sin 10 = 1 \times 9.8 \times 10^{-9}$$

$$\therefore d = 2.8 \times 10^{-8} m$$

Hence, the spacing between two principle planes of crystal is $2.8 \times 10^{-8} m$

Q. Find the atomic number of element that has K_β x-ray of delivery

Soln,

for K_β line

$n_1 = 1$

$n_2 = 3$

we know,

$$f = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or, } \frac{c}{\lambda} = RcZ^2\left(\frac{1}{1} - \frac{1}{9}\right)$$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{8}{9}\right)$$

$$\therefore Z = \sqrt{\frac{9}{8R\lambda}}$$

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Q The wavelength of l_α line of x-ray in the case of Platinium $z=78$ is 1.321 A^0 and unknown substance emits l_α line of wavelength 4.174 A^0. Calculate the atomic number of unknown substance given that $b=7.4$ for l_α line

Soln,

we know that

$$\frac{1}{\lambda_1} = Ra(z-b)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \text{---(i)}$$

and, $$\frac{1}{\lambda_2} = Ra(z_2-b)\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \text{---(ii)}$$

dividing (i) and (ii)

$$\frac{\lambda_2}{\lambda_1} = \frac{(z-b)^2}{(z_2-b)^2}$$

$$\Rightarrow \frac{4.174}{1.321} = \frac{(78-7.4)^2}{(z_2-7.4)^2}$$

$$\Rightarrow (z_2-7.4)^2 = 1577.48$$

$$\Rightarrow z_2-7.4 = 39.72$$

$$\Rightarrow z_2 = 47.12$$

Nuclear transformation:

Radioactivity:

Radioactivity is a process in which α, β and γ radiations are emitted from unstable nucleus. It is fully natural process. It cannot be the artificial means. The examples of radioactivity are Radium, Polonium, Uranium, Platinium.

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α-radiation	β-radiation	γ-radiation
Ionized He	Electron	Electromagnetic wave
+ve charge	-ve charge	No charge
mass = $4m_p$	mass = m_e	No rest mass
charge = Ze	charge = e	No charge
very high ionization power	less ionization power	very low ionization power
very low penetrating power	low penetrating power	very high penetration power
very low speed	speed = speed of free electron	speed = speed of light

Electromagnetic wave:
Radio
micro
Infra
visible
UV
x-ray
γ-ray

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Sucessive radioactive disintegration:-

It is a chain radioactive disintegration in which first sample of a radioactive source disintegrates to form a second sample. The second sample also disintegrates to form third sample. The process is continue until the stable nucleus is formed.

Example $A \rightarrow B \rightarrow C \rightarrow$ - - - - (Sample nucleus)

Suppose we have a samples A, B and C of radioactive source. The sample A disintegrates to form sample B and the sample B also disintegrates to form sample C. which is stable

Now,

At time $t=0$,

No of atoms in sample $A = N_0$

No of atoms in sample $B = 0$

At $t = t$

No of atoms in sample $A = N_1$

No of atoms in sample $B = N_2$

Decay of atom A (formation of B) $= \lambda_1 N_1$

and decay of $B = \lambda_2 N_2$

where, λ_1 and λ_2 are decay constants for A and B respectively

$$\boxed{N = N_0 e^{-\lambda t}}$$

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The increase in atoms in the sample B at time t is

$$\frac{dN_2}{dt} = (+N_1\lambda_1 - N_2\lambda_2)$$

$$\Rightarrow \frac{dN_2}{dt} + N_2\lambda_2 = N_1\lambda_1$$

$$\Rightarrow \frac{dN_2}{dt} + N_2\lambda_2 = N_0 e^{-\lambda_1 t} \cdot \lambda_1$$

multiplying both sides by $e^{\lambda_2 t}$

derivation

$$\Rightarrow \frac{dN_2}{dt} \cdot e^{\lambda_2 t} + N_2 \lambda_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{-\lambda_1 t} e^{\lambda_2 t}$$

$$\Rightarrow \frac{d}{dt}\left(N_2 e^{\lambda_2 t}\right) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

On integrating,

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_0 \, e^{(\lambda_2 - \lambda_1)t}}{(\lambda_2 - \lambda_1)} + c$$

At $t = 0$

$N_1 = N_0 \qquad N_2 = 0$

$$\frac{\lambda_1 N_0}{\lambda_1 - \lambda_2} = c$$

substituting c in above equation,

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$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_0}{(\lambda_2-\lambda_1)} e^{(\lambda_2-\lambda_1)t} - \frac{\lambda_1 N_0}{(\lambda_2-\lambda_1)}$$

$$\Rightarrow \boxed{N_2 = \frac{\lambda_1 N_0}{(\lambda_2-\lambda_1)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]}$$

This is expression for number of atom in Sample B at time t.

EQULIBRIUM:-

There are two kinds of equilibrium incase of sucessive radioactive disintegration

secular equilibrium (Permanent equilibrium)

) Transient equilibrium

Secular equilibrium:

In case of secular equilibrium, parent nucleus is infinitely long lives incompare to the daugther nucleus. That means the half life of parent nucleus is very very higher than the half life of daughter nucleus.

let T_1 is half life of parent nucleus (A) and T_2 be that of B

Here, $T_1 >> T_2$

$$\lambda_1 = \frac{0.693}{T_1} \to 0$$

$$\lambda_2 = \frac{0.693}{T_2} = \text{very large}$$

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Then the above expression becomes

$$N_2 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}\left(1 - e^{-\lambda_2 t}\right)$$

After a very long period of time,

$N_0 = N_1$

Then,

$$N_2 = \frac{N_1 \lambda_1}{\lambda_2}(1-0) \qquad (\lambda_1 \to 0)$$

$$\boxed{\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2}}$$

This is required equation for secular equibrium.

• Transient equilibrium:-

In this type of equilibrium parent nuclei is long life as compare to daughter nucleus i.e half life of parent nucleus is higher than the half life of daughter nucleus.

$$T_1 > T_2$$

$$\rightarrow \lambda_1 = \frac{0.693}{T_1} = \text{small}$$

$$\lambda_2 = \frac{0.693}{T_2} = \text{large}$$

Then expression for N_2 becomes

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

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$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(1 - e^{-\lambda_2 t}\right)$$

After long time,

$$N_0 = N_1$$

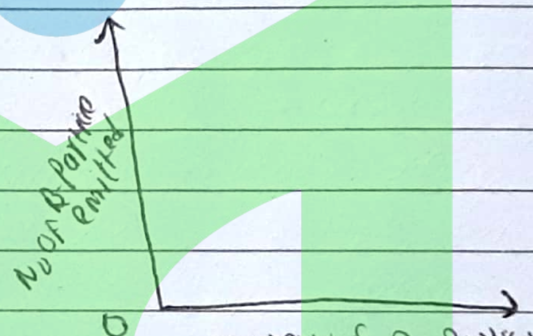
Then,

$$N_2 = \frac{\lambda_1 N_1}{\lambda_2}(1-0)$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}}$$

This is equation of transient equilibrium

β-ray spectrum



Initially it was assumed that there is emission of electron only during β-decay. Experimentally it is found that β-ray spectrum is continuous spectrum having a peak value of energy as shown in figure. The graph clearly suggested that the β-ray spectrum is continuous and the β-particles emitted from radioactive nucleus having large range of energy. A large number of β-particles have low energy and few of β-particles have maximum energy.

To explain β-ray spectrum, we have to consider mass energy relation. Let m_1 be the mass of parent nucleus and m_2 is that of daughter nucleus, then according to Einstein mass energy relation,

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$$(m_1 - Zm)c^2 \rightarrow (m_1 - (Z+1)m)c^2 + {}_{-1}e^0 + Q$$
$$(mc^2)$$

This equation shows that the difference in rest mass energy between mother and daughter nuclei is the energy carried by β-particle. Experimentally it is found that, the energy carried by β-particles is less than the above mentioned value. The above equation doesnot hold the ~~cons~~ conservation of spin as well as linear momentum. For this purpose a new elementary particles called neutrino emits with β-particles. The equation of emission of β-particle with neutrino preserve the conservation of linear momentum and spin momentum.

[Neutrino theory of β-decay] | Fermi theory]

During the β-decay not only the β-particles but also a massless particles called neutrino emits simultaneously. The energy released due to the nuclear reaction is divided into the energy of residual nucleus, energy of β-particles and energy of neutrino.
If the β-particles carries maximum energy, then the neutrino carries minimum energy and vice-versa due to the sharing of energy.

$h\nu$

between β-particles and neutrino, the β particles can have all possible range of energy. Hence, β-ray spectrum is continuous

When the nucleus goes from neutron quantum state to proton quantum state, there is a emission of β particle along with antineutrino.

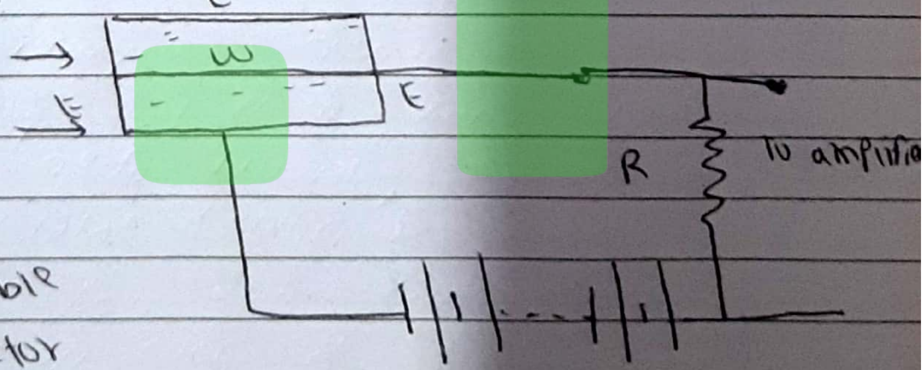
$$_0n^1 \longrightarrow {}_1p^1 + \underset{\beta}{({}_{-1}e^0)} + \bar{\nu}$$

Similarly, when proton quantum state goes to neutron quantum state, then there is emission of position along with neutrino as in the given nuclear reaction

$$_1p^1 \longrightarrow {}_0n^1 + {}_1e^0 + \nu$$

In 1934, fermi developed a theory to explain continuous β-ray spectrum. This theory is called neutrino theory of β-decay.

GM Counter:-



for the detection of nuclear radiation a large number of devices are available they are called detector out of them geiger muller (GM) counter is simplest one

fig: GM counter

Construction:

The GM counter consists of metal chamber (enclosing air or some gases at a pressure about 10cm of Hg, at the middle of chamber a tungsten wire 'W' is stretched along the length of the chamber. The chamber and the wire are insulated from Ebonide plug 'EE'. The wire is connected to a +ve potential of high voltage power supply through a very high resistance of which is of the order of megaohm (MΩ) and the chamber is connected to negative terminal of high voltage battery. There is a window at the back side that allow the radiation to enter inside the chamber.

Working:

When a power is switched on the wire at at +ve potential and chamber is at -ve potential when an ionizing particle (α-particle) enter into the chamber it ionises the gas and few ions are produced. The ions are accelerated in the electric field by the collision they produce further ions. As a result avalanche of electrons moves towards the wire 'W'. When electrons reach the wire they flow through the resistance R. Hence, the voltage across R

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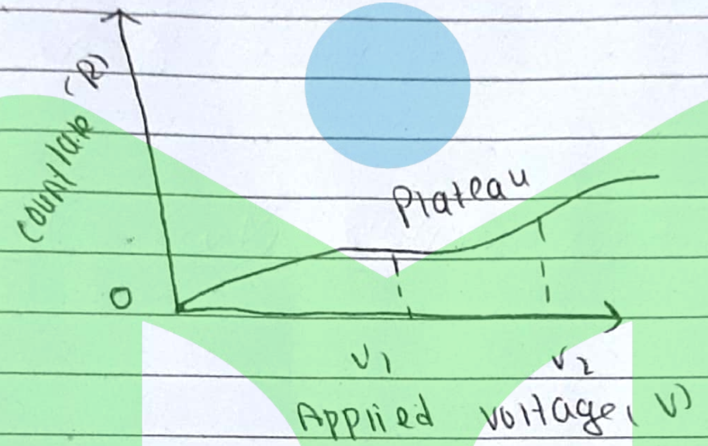
appears in the form of pulse which can be amplified by an amplifier and then recorded by the counter. Each pulse across 'R' causes reading of the counter to increase by one. Hence in this way, the ionizing particles are detected.

Plateau of GM counter.



If we draw a graph between applied voltage and counter rate (no of count per min or sec). Applied voltage across the wire 'W' and the chamber of GM counter (tube) is below a certain value called threshold voltage. No ion can reach the wire. As a result the counter reads zero. As the voltage goes increasing more and more ions are created in the chamber as result counting increases. Further increase in voltage doesnot increase the count rate ie the curve is flat. The flat portion of VR is called plateau. If the applied voltage increases beyond the Plateau the count rate increases rapidly.

Working voltage.

The best working voltage of the GM counter is the voltage which is gives the true count. That why we can choose the working voltage as a voltage corresponding to plateau region

i.e.

$$\text{Working voltage } (V_w) = \frac{V_1 + V_2}{2}$$

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Particle accelerator:-

Accelerator is a device which is used to accelerate charge particle after acceleration, the charge particle gets maximum kinetic energy. The uses of particle accelerator are:-

i) They are used in the field of research.

ii) They are used as the basic component in the high energy particle physics.

iii) The particle having very high kinetic energy obtained from accelerator can be used to break the nucleus. As a result, large amount of energy is released which can be used for distruction purpose.

There are two kind of accelerator:-

i) linear accelerator   ii) cyclotron accelerator

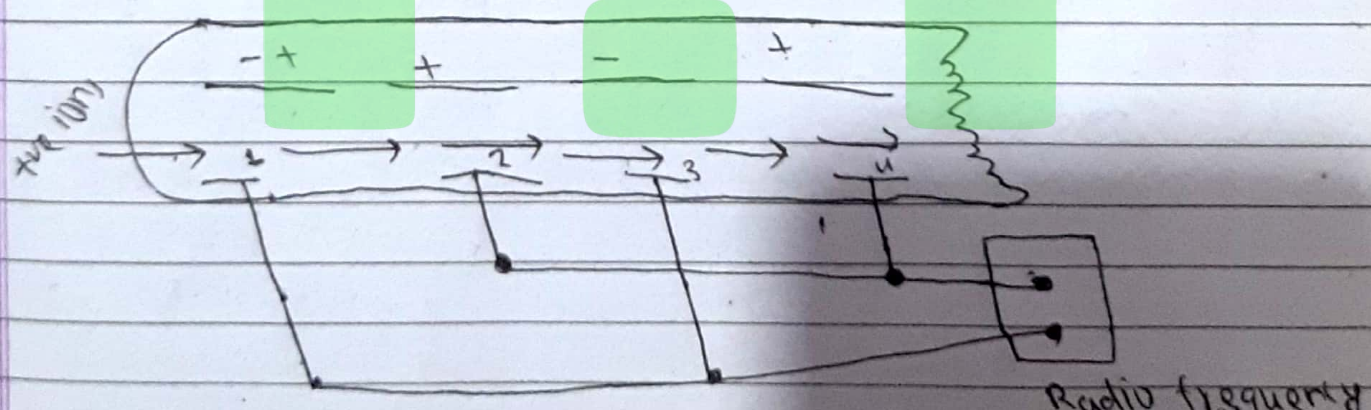

fig: linear accelerator

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Construction:-

It consists of hollow metalic cylinder which is called drift chamber. Inside the drift chamber there are large number of metalic cylindrical tubes. They are called drift tube. The odd number of tube are connected to one terminal of high frequency radio oscillator. While the even number of tubes are connected to another terminal of the oscillator. The ion source is placed infront of first tube. The length of tube and gap between tube are successively increased. These is placed inside chamber which is completely vacant.

The positive ions are shooted at a time during which first tube become negative. The ion get accelerated within a gap and get maximum kinetic energy. i.e

$$qv = \frac{1}{2}mv^2 \longrightarrow (i)$$

Here,
$q \rightarrow$ ionic charge
$v \rightarrow$ accelerating potential
$m \rightarrow$ mass of an ion
$v \rightarrow$ velocity of ion near first tube.

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from, eq^n (i)

$$v_1 = \sqrt{\frac{2qV}{m}} \quad \text{—(ii)}$$

The ion are not accelerated within a tube because the tube are field free region whenever the ions comes out from first tube, the ~~sa~~ cycle applied source changing such that second tube become negative. Again the ions get accelerated within the gap between first and second tube. let v_2 be the velocity of ion ~~e~~ in between 1st and 2nd gap. then,

$$v_2 = \sqrt{2}\, v_1 \quad \text{—(iii)}$$

let v_n be the velocity of ion on crossing nth tube.

Then by the relation

$$v_n = \sqrt{n}\, v_1 \quad \text{—(iv)}$$

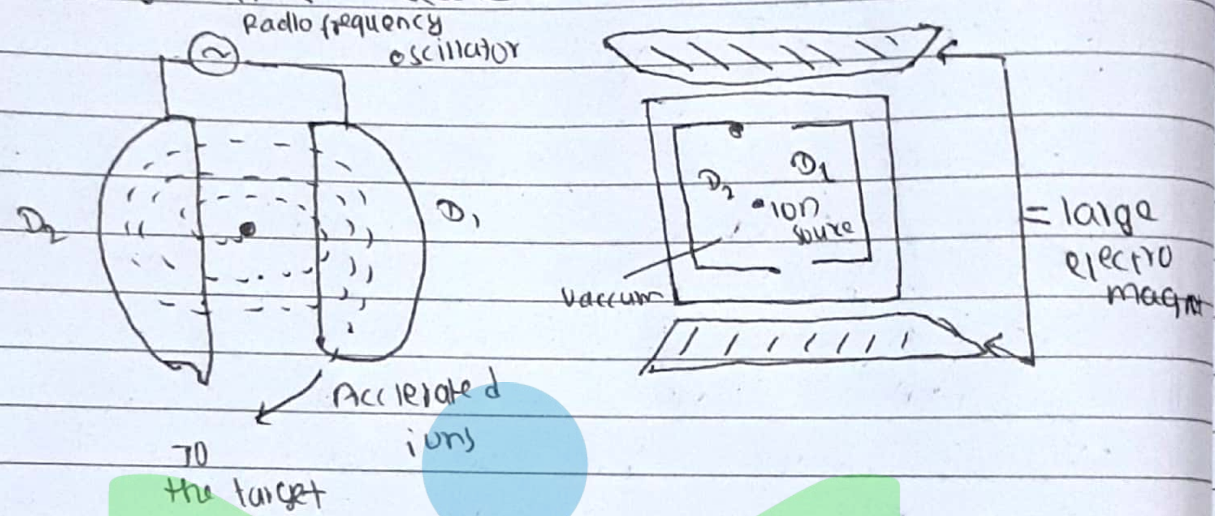
finally kinetic energy gain by ions on crossing nth tube,

$$KE = \frac{1}{2} m v_n^2 = \frac{1}{2} m n v_1^2 = \frac{1}{2} n m v_1^2$$

$$\boxed{KE = nqV} \quad \text{—(v)}$$

Hence, from equation (v) energy of ion coming from nth tube is directly proportional to the number of tube used and energy supplied to first tube.

ii) cyclotron accelerator



Construction:-
It consists of two semi-conductor metallic disc disc. D_1 and D_2 called dees. In between these two dees an ion source is placed and the two dees are connected to a high frequency radio oscillator. And the dees are placed inside the vaccum chamber. The two poles piece of electromagnet are placed on the either side of dees to produce perpendicular magnetic field on the plane of dees.

Working:-
When the positive ions is ~~shift~~ shooted from the gap between two dees, the disc, t is negatively charged. The ion get accelerated within the gap and the magnetic field deviate the particles along the circular path of radius 'r'.

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If B be the strength of magnetic field and 'v' be the velocity of ion then we have,

$$\frac{mv^2}{r} = Bqv$$

$$\Rightarrow mv = Bqr$$

$$\Rightarrow m\omega r = Bqr$$

$$\Rightarrow m \cdot \frac{2\pi}{T} = Bq$$

$$\Rightarrow \boxed{T = \frac{2\pi m}{Bq}}$$

Now,
time taken to travel semicircular path,

$$\boxed{T = \frac{\pi m}{Bq}} \quad \text{--- (i)}$$

From the equation (i) it is cleared that the travelling ion on a circle of greater radius, the time required to travel circular path always remain constant, but the velocity of ion increases on a outer circular orbit. By this process, the ion travels number of circular path getting higher and higher kinetic energy. Let r_n be the radius of 'n' circular path describe by the ion and its KE is given by,

$$KE = \frac{1}{2} m v_n^2$$

$$\frac{mv_n^2}{r} = Bqv_n$$

$$\Rightarrow v_n = \frac{Bqr}{m}$$

$$\Rightarrow KE_n = \frac{1}{2} m \left(\frac{B^2 q^2 r_n^2}{m^2}\right)$$

$$\therefore KE_n = \frac{1}{2m} B^2 q^2 r_n^2$$

Here, energy of ion coming out from n^{th} circular orbit depends upon the square of radius of that orbit. This means on increasing the radius of orbit kinetic energy of ion get increased as a result the ion get accelerated.

Limitation :-

From the cyclotron, we can obtain the ion velocity which is comparable to speed of light. This indicates that there must be relativistic variation of mass of ion with velocity.

$$t = \frac{\pi m}{qB}$$

$$t = \frac{\pi m_0}{qB\sqrt{1-\frac{v^2}{c^2}}}$$

where $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is relativistic mass

total time to complete circular orbit,

$$T = \frac{2\pi m_0}{qB\sqrt{1-\frac{v^2}{c^2}}}$$

from these equation it is cleared that the velocity of ion increases on increasing frequency. It means ion takes longer time to complete the longer path circular path of higher radius. Thus, there may be the chance of decreasing the energy of ion. This limitation can be removed either of then two way.

i) Frequency variation
ii) Field variation

(A) Detector:-
i) Bubble chamber
ii) crenkov dectector
iii) ~~ce~~ sintillination counter
~~iv)~~ ~~ioo~~

(B) Large hadron collider (LHC)

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* In cyclotron, the maximum magnetic field of orbit was 0.4 T operating at 50 Hertz with the diameter of 1.52 m. calculate the average energy gain per revolution by an electron?

$$E = \frac{1}{2m} B^2 q^2 r_n^2 \qquad m = 9.1 \times 10^{-31}\,kg$$

$$E/f = \frac{\frac{1}{2m} B^2 q^2 r_n^2}{50}$$

$$= \qquad J/Hz$$

* In a Betatron the maximum magnetic field of orbit was 0.4 T operating at 50 Hertz with

* ~~proton~~ Deutron in cyclotron describe the circle of radius 0.32 m just emerging. The frequency of applied emf is 10 MHz find the flux density of magnetic field emerging out of cyclotron

$$Bqv = \frac{mv^2}{r}$$

$$\therefore B = \frac{mv}{r \cdot q} = \frac{m\omega r}{rq} = \frac{2\pi f m}{q}$$

$$= \frac{2\pi \times 10^7 \times 3.32 \times 10^{-27}}{1.6 \times 10^{-19}}$$

$v = 2\pi f r$

* A cyclotron with dees of radius 90 cm has a transverse magnetic field of 0.8T. calculate the energy to which proton and deutron are accelerated

* 0.55 mev protons are injected into a 55 meV linear accelerator powered by 200 MHz radio frequency supply find the approximate length of last drift tube.

$$v = \sqrt{\frac{2E}{m}} \rightarrow 55\,meV$$

$m = 1.6 \times 10^{-27}$

$$T = \frac{1}{f}$$

Appr. length of last drift tube $= \frac{T}{2} \times v$